Response to Comment on "How Cats Lap: Water Uptake by *Felis catus* "
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Science **334**, 311 (2011);
DOI: 10.1126/science.1203157

Response to Comment on "How Cats Lap: Water Uptake by *Felis catus*"

Roman Stocker,[1]* Jeffrey M. Aristoff,[2] Sunghwan Jung,[3] Pedro M. Reis[1,4]

We return to the physics of cat lapping to show that our proposed scaling analysis predicts the functional dependencies revealed by the experimental data more accurately than a recently proposed alternative description by Nauenberg. Experimental verification of functional dependencies, rather than single numerical values, represents the appropriate test for any scaling argument.

We reported observations, laboratory experiments, and scaling analyses revealing that cats drink by creating a column of liquid with their tongue and that the column's dynamics are controlled by the competition of gravitational and inertial forces (*1*). Here, we address a comment by Nauenberg (*2*), who argues that our analysis is incorrect. We show that Nauenberg's claim is erroneous and conflicts with the good agreement between our scaling predictions and experimental data, which his alternative theory fails to achieve.

The deceivingly simple act of lapping conceals complex fluid mechanics, involving a free surface and a boundary (the tongue) traveling at variable speed. Scaling is a classic approach to gain insight into such complex flows: One renounces an exact solution and instead identifies the dominant ingredients in the equations of motions to uncover fundamental mechanisms and predict dependencies between observables and parameters of the flow. Numerous problems in fluid mechanics, from the drag on a sphere (*3*) to the energy in a nuclear blast (*4*), can be approached by scaling analyses. Inherent in this method is the acceptance that it cannot predict exact numerical values.

We used scaling to understand lapping. We neglected inertia in the vertical momentum equation, thereby assuming a hydrostatic pressure distribution, which led to a balance between pressure and inertia in the radial momentum equation. This yielded a characteristic time scale for lapping $t_{RJAS} = R/(gH)^{1/2}$, where R and H are the radius and final elevation of the tongue and g is the acceleration of gravity (*1*). In contrast, Nauenberg asserts that the relevant time scale is the time of free fall of an object in vacuo, $t_G = (H/g)^{1/2}$. We here evaluate this alternative hypothesis in light of our data.

[1]Department of Civil and Environmental Engineering, Massachusetts Institute of Technology (MIT), Cambridge, MA 02139, USA. [2]Department of Mechanical and Aerospace Engineering, Princeton University, Princeton, NJ 08544, USA. [3]Department of Engineering Science and Mechanics, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061, USA. [4]Department of Mechanical Engineering, MIT, Cambridge, MA 02139, USA.

*To whom correspondence should be addressed. E-mail: romans@mit.edu

Let us consider a liquid column collapsing under its own weight. Despite the fundamental nature of this problem, little is known about the collapse time scale. An important result dates back to Martin and Moyce's (MM) experiments (*5*), which revealed that the collapse time scale for an initially confined column is $t_{MM} = (R/g)^{1/2}$. This time scale is intermediate between $t_G = t_{MM}\,(H/R)^{1/2}$ and $t_{RJAS} = t_{MM}\,(H/R)^{-1/2}$.

These three time scales—t_G, t_{MM}, and t_{RJAS}—lead to different scaling predictions for an important parameter that we quantified systematically in our laboratory experiments: the height of the column at break-up, Z_P. If the collapse time scale is t_{RJAS}, the column reaches a height $Z_P \sim t_{RJAS}U$ before breaking up, where U is the speed of the tongue [in contrast with (*1*), here we denote U_{MAX} by U]. For this scenario, we have $t_{RJAS} = R/(gZ_P)^{1/2}$, yielding $Z_P/R \sim \mathrm{Fr}^{2/3}$, where $\mathrm{Fr} = U/(gR)^{1/2}$ is the Froude number. If the collapse time scale is instead t_{MM}, one predicts $Z_P \sim t_{MM}U$ and thus $Z_P/R \sim \mathrm{Fr}$. Finally, if the collapse time scale is t_G, we can write $t_G \sim (Z_P/g)^{1/2}$, because the column has height Z_P when it breaks, and this yields $Z_P/R \sim \mathrm{Fr}^2$. Contrasting these three scaling relations with experimental data (Fig. 1) reveals that our prediction closely matches the observations, and the prediction stemming from t_{MM} somewhat overpredicts their slope, whereas the prediction based on t_G is considerably awry. This detailed comparison lends strong support to our scaling analysis and illustrates that Nauenberg's choice of time scale is refuted by the data.

We further validated our scaling analysis by considering the height of the column at which the column's volume is maximal, Z_{MaxVol}. This quantity is important because it enters our argument that cats drink in a manner that maximizes the ingested volume (*1*). Nauenberg's argument does not permit a prediction of Z_{MaxVol}, but we highlight that our scaling prediction—$Z_{MaxVol}/R \sim \mathrm{Fr}^{2/3}$ (*1*)—is in good agreement with observations [see figure 4A in (*1*)]. We note that, although Z_{MaxVol} and Z_P exhibit the same scaling with Fr, they provide separate tests for the theory because they were independently determined from the experiments.

The success of our scaling for Z_P also corroborates our conclusion that capillary forces are, to first order, unimportant in lapping (*1*). Whereas the final stages of the column's shrinking and consequent pinch-off are clearly driven by surface tension and a Rayleigh-Plateau–like instability, capillary forces only become comparable to gravity (i.e., the Bond number is of order 1) (*6*) when the radius of the column is <2 mm. Thus, the majority of the contraction is driven by gravity.

Nauenberg does not provide the reasoning underlying his choice of time scale, but we speculate that it could have two origins. If t_G results simply from porting an elementary physics result (an object's free-fall time, as suggested by the factor of $\sqrt{2}$ in Nauenberg's expression for t_G) into this fluid-dynamical process, then it should equally apply to Martin and Moyce's experimental results. The latter, however, revealed a different time scale, $t_{MM} \sim (R/g)^{1/2}$: A liquid column does not fall freely, because fluid parcels hinder one another and the collapse time depends on the radius of the column. The validity of Martin and Moyce's findings for columns as slender as $H/R = 4$, close to the lapping regime ($H/R = 6$),

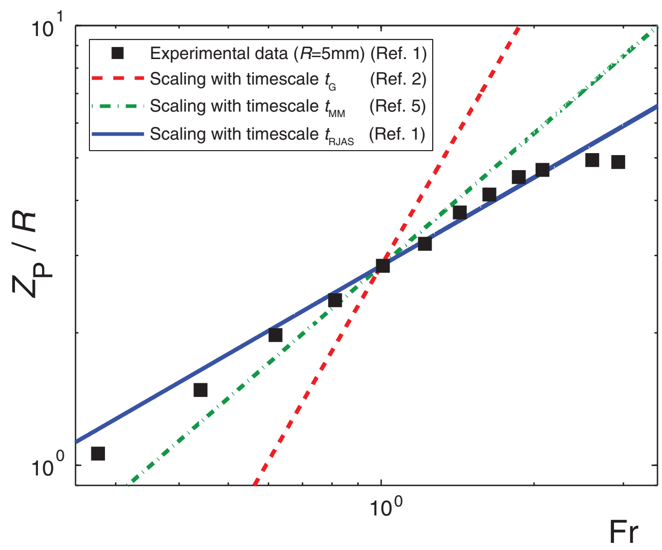

Fig. 1. Comparison of theoretical scaling relations and experimental data for the height of the column at pinch-off, Z_P. The three time scales t_G (*2*), t_{MM} (*5*), and t_{RJAS} (*1*) yield three scaling predictions for the functional dependence of Z_P/R on the Froude number, Fr. The experimental data are from figure 4A in (*1*) and correspond to a disk of radius R = 5 mm (closest to the parameters corresponding to the domestic cat) raised from a liquid bath at different speeds.

further refutes t_G as the relevant collapse time scale.

Alternatively, t_G can be obtained through a scaling analysis of the momentum equations for slender columns ($H/R >> 1$). If this is Nauenberg's rationale, then his subsequent quest for a numerical comparison with a single data point—in lieu of a test of functional dependencies—is unwarranted, because $(H/g)^{1/2}$ is a scale for the collapse time, not a numerical estimate of it. Furthermore, Nauenberg's quantitative comparison itself is tainted by the introduction of numerical factors ($\sqrt{2}$ in t_G and ¼ in the lapping frequency), conveniently limited to his own estimate and unorthodox in scaling. Likewise, for dogs, Nauenberg assumes an unsupported lapping height of 6 cm followed by a comparison with a single data point, again undermining the essence of scaling.

Instead, a scaling analysis is successful when it predicts functional dependencies, as ours does (Fig. 1). This represents a considerably stronger test than comparing a single lapping frequency, because numerical coefficients remain undetermined in scaling analyses. Whereas these coefficients are often of order 1, they can occasionally be large, as in the case of the drag on a sphere at low Reynolds numbers, where the coefficient is 6π (*3*), or the instability of a circular hydraulic jump, where it is 74 (*7*).

Underlying our scaling is the assumption that pressure is hydrostatic, whereas the scaling leading to t_G assumes a uniform pressure along the column. A priori, it is difficult to determine which hypothesis is correct. On the one hand, t_G can be derived assuming $H/R >> 1$, potentially adequate for cats ($H/R = 6$). On the other hand, the assumption of hydrostatic pressure is appropriate for thick columns (*8*, *9*), and in lapping the column is initially thick. The extent to which pressure relaxes from hydrostatic to uniform as the column is stretched remains unknown, although comparison with data favors our assumption.

In closing, we emphasize that our central result—that lapping in cats is governed by the balance of inertial and gravitational forces (*1*), thus belonging to a wider class of Froude mechanisms in biology (*10*)—remains unchanged by this debate, which revolved around the specific form that inertia assumes in the equations of motion. A full numerical solution, which we hope our work will spark, can conclusively settle this issue. Until then, as with many problems in fluid mechanics, scaling and systematic testing of functional dependencies against experimental observations remain invaluable tools to discern among disparate hypotheses.

References and Notes

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16 February 2011; accepted 22 September 2011
10.1126/science.1203157